The Impact of Element Taper and Inhomogeneous Material Properties on Ultrasonic Array Performance

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Abstract - The computer simulation of an ultrasonic transducer is typically an idealized representation of the device and assumes perfectly parallel faces and spatially invariant material properties. In the real world however, these assumptions are not necessarily valid. Fabrication processes such as dicing may result in a quantifiable taper across the height and/or length of a transducer array element. Furthermore, material properties, in particular those for piezoelectric materials, may vary appreciably across the length of an array element. Both of these "parasitic" facts-of-life will impact a device's performance to some extent or another, and in some cases may serve to degrade device performance below acceptable limits. The purpose of this paper is to study and quantify the impact that non-idealized geometries and inhomogeneous material properties have on an ultrasonic transducer's resonant response.

I. Introduction

The latest version of PZFlex [1], an explicit time-domain finite element code, now permits a skewed coordinate system and hence conformal elements to be used with piezoelectric materials. This enhanced capability facilitates the analysis of transducer structures that suffer from tapered sides due to the inherent limitations of standard fabrication processes. This class of structure would not have been readily analyzable via a traditional cartesian grid.

In this paper, we will utilize the skewed grid capabilities of PZFlex to analyze both 1-D and 2-D array elements that have tapered sides or length. Furthermore, we will also explore what happens as we change the material properties across the width of a 1-D array element. Simulated results will be compared with quantifiable experimental data. Based on the results of the analysis, a set of recommendations and observations will be produced that may be used by a transducer designer and modeler to determine the most appropriate model for a given design task. These results will also help identify acceptable structural and material variability tolerances for a given array application.

II. Modeling Background

A question that is often asked when a transducer designer starts using advanced modeling tools for the first time is "What level of correlation should I be striving for?" The purpose of this paper is to try and quantify the difference between simulated and experimental results when structural details and/or material properties differ from nominal values. The modeler should then be better equipped to decide what level accuracy is required for his/her design application and the cost-benefit trade off.

PZFlex is an explicit time-domain finite element code that has been developed to facilitate high-speed analysis of both linear and non-linear wave propagation in piezoelectric, electrostatic, electrostrictive and magneto-strictive applications. The highest computational performance is achieved when the finite element model is constructed using an orthogonal cartesian grid, otherwise known as a *standard partition*. Typically, this also results in a model with minimum memory requirements. The alternative approach is to use a skewed grid (*skewed partition*) that allows the finite elements to be distorted to better match the physical structure. Skewed grids require greater memory storage and take longer to compute than their cartesian counterparts (on a per element basis). In many cases, it is possible to conform to a given curvature by stair-stepping a cartesian grid with appropriately sized elements and still have a calculation that runs faster than it would have done with skewed elements.

There are certain applications however where a stair-stepped approximation is not sufficient to accurately represent model geometry without making the elements unacceptably small. Under these circumstances the rational way forward is to use skewed elements. Previous versions of PZFlex supported skewed element geometries for passive material types; however, the latest version of PZFlex (1.j.7) also supports skewed piezoelectric elements.

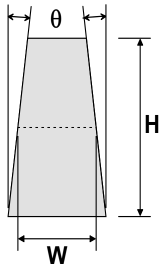

Figure 1: Trapezoidal element shape

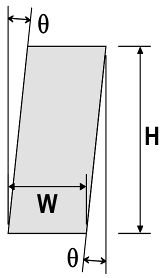

Figure 2: Parallelepiped element shape

Accurate specification of a transducer's physical structure and all the materials used is imperative if accurate results are to be obtained. However, it is the skill of the transducer designer to discern the level of accuracy that is required for a particular design application. For example, general design/engineering decisions can often be made via simpler models and it is not necessary to spend the additional time and effort that would be required to configure a skewed model and incur the associated increased memory requirements and the possibility of an extended compute time.

The objective of this paper is to identify the level of correlation that may be obtained via stair-stepped models using homogenous material properties. The results of this analysis will allow us to identify the limitations of a cartesian grid and determine when it becomes necessary and/or advantageous to adopt a more comprehensive skewed finite element representation for the device of interest.

III. Demonstration of Improved Correlation With Experimental Data

Anybody that has been actively involved in the numerical simulation of real-world systems will be quick to recognize that the closer a numerical model is to reality, the better the correlation will be between experimental and simulated data. Consequently, this requires accurate quantification of both material properties and structural geometry.

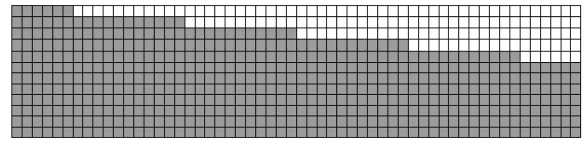

Figure 3: Regular stair-stepped mesh

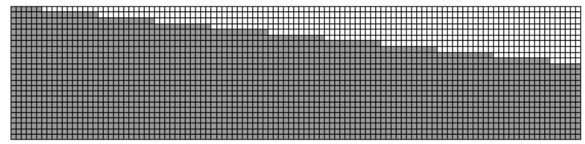

Figure 4: Fine stair-stepped mesh

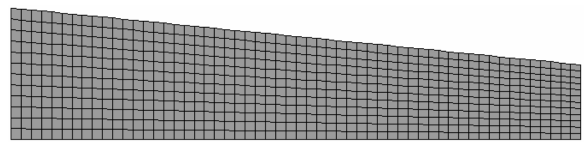

Figure 5: Skewed conforming mesh

The two resonator geometries shown in Figures 1 and 2 were cut from a plate of Motorola 3203HD with a view to demonstrating the improved level of correlation and improved computational efficiency that may be obtained via the appropriate application of skewed finite element models. Since it is not possible to easily quantify the spatial variation of material properties along the length of an element or through its width & height, we have assumed homogenous material properties for the time being. Subsequent sections of this paper will take a closer look at the effects of spatially variant material properties.

Height Taper – The Trapezoid

Two examples of the trapezoidal configuration (Figure 1) were fabricated, with widths (W) 0.5mm and 0.75mm. In both cases, the elements measured 1.53mm high by 30.07mm long and had a taper angle (θ) of 5°. The devices were simulated using each of the finite element meshes shown in Figures 3, 4 & 5. Furthermore, a non-tapered cartesian mesh was also included for reference purposes. The fine cartesian stair-stepped mesh (Figure 5) shows how it is possible to better represent the element's tapered sides by adopting a smaller element size. However, this comes at the expense of greater memory requirements and a smaller time step (i.e. more time steps are required for the same overall run length). Even though a skewed finite element model does have increased computational requirements over that of a conventional cartesian grid on a per element basis, it can still prove to be more efficient solution for applications that would have otherwise necessitated a very fine cartesian grid for accurate analysis. The results for the 0.5mm wide

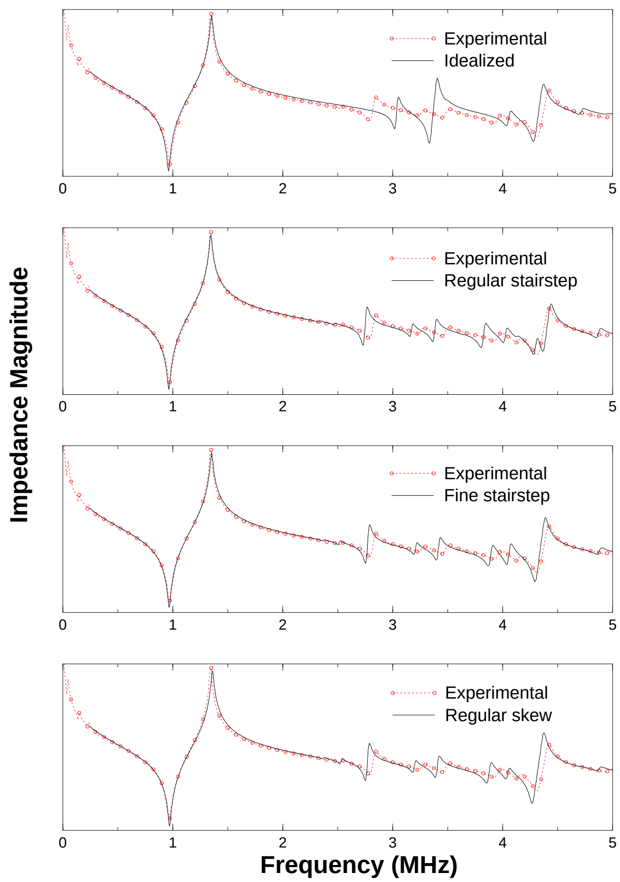

Figure 6: Impedance response for 0.5mm wide trapezoid

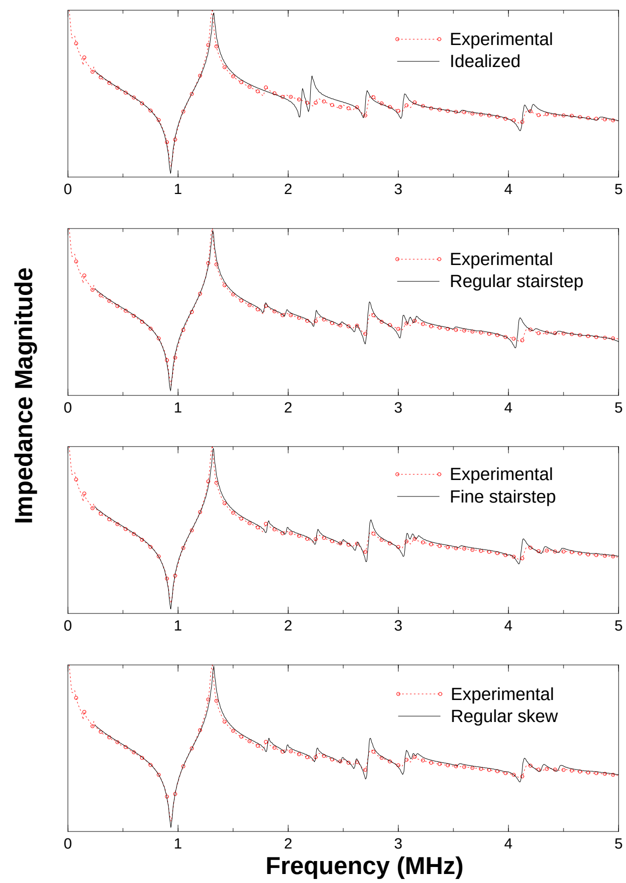

Figure 7: Impedance response for 0.75mm wide trapezoid

trapezoid are shown in Figure 6. It can be seen that as the finite element grid is refined from the non stair-stepped idealized case, through a regular stair-stepped configuration and ultimately onto a finely discretized stair-stepped model, the level of correlation between experiment and simulation is incrementally improved. This is especially true at the higher frequency resonances that correspond to lateral modes within the resonator and higher harmonics of the fundamental resonance at 1MHz. It should also be noted that the regular stair-stepped grid incorrectly predicts an "additional" resonance at just over 4MHz. The skewed result (bottom) is virtually identical to that obtained via the fine stair-stepped model. Furthermore, due to the significantly reduced number of elements, the skewed model ran four times faster than the fine mesh model. In other words, the same level of accuracy (or perhaps even better) is possible via a model that runs in a quarter of the time. It is important to note that the improved level of correlation we are currently seeking relates to the more accurate representation of lateral and higher order modes. Correlation at the primary resonance is not an issue, since even 1D equivalent parameter modeling techniques can be made to reliably predict those resonances.

Figure 7 shows the 0.75mm wide case. In this case, all of the conforming models, whether they are skewed or stair-stepped, show good agreement with the experimental data, and it may be seen that it is not as important to adopt a more accurate mesh representation as in the previous example. This may be explained as follows. In all cases the primary resonance has been accurately predicted (even with idealized geometry) since it is primarily dependant upon the height of the resonator. The higher resonances (>2.5MHz) however, are lateral modes in the device and higher harmonics of the primary resonance. These resonances are more sensitive to changes in the element's width. For a given taper angle, the relative change in element width from the top of the resonator to its bottom, is greater for narrow elements than for wider elements, and hence their increased sensitivity to taper. Consequently, greater care must be taken when working with narrower array elements to ensure acceptable element to element uniformity.

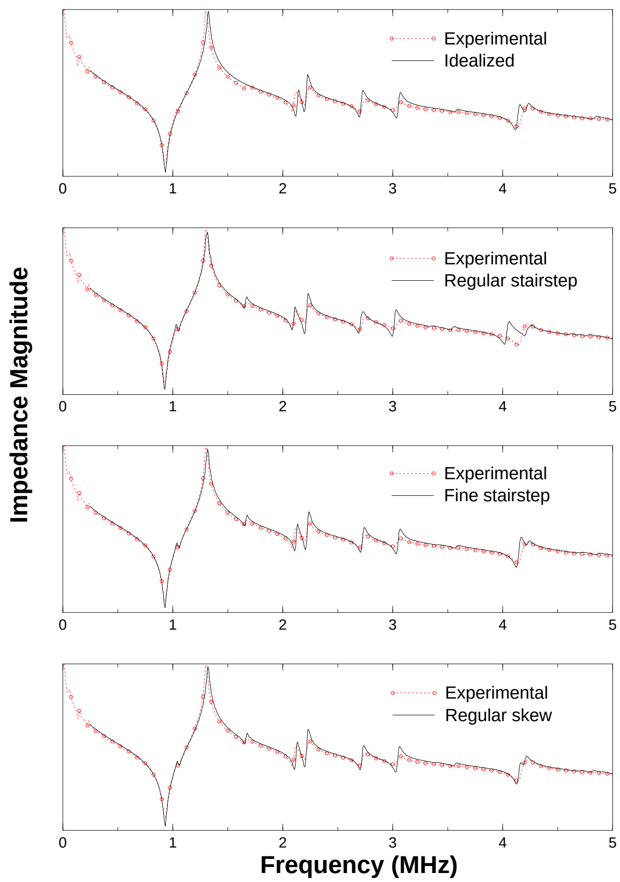

Figure 8: Impedance response for parallelepiped

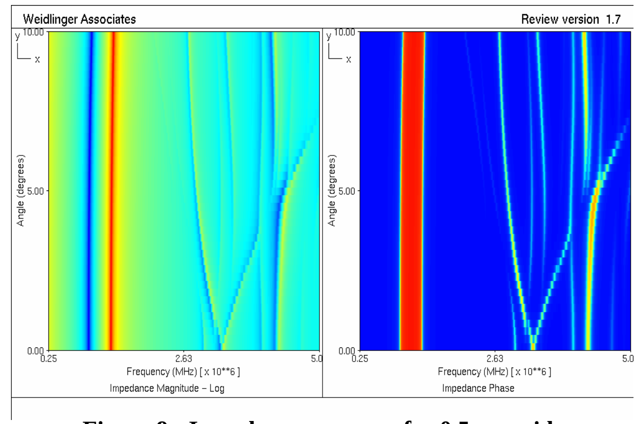

Figure 9: Impedance response for 0.5mm wide trapezoid (|Z| - left, ∠Z – right)

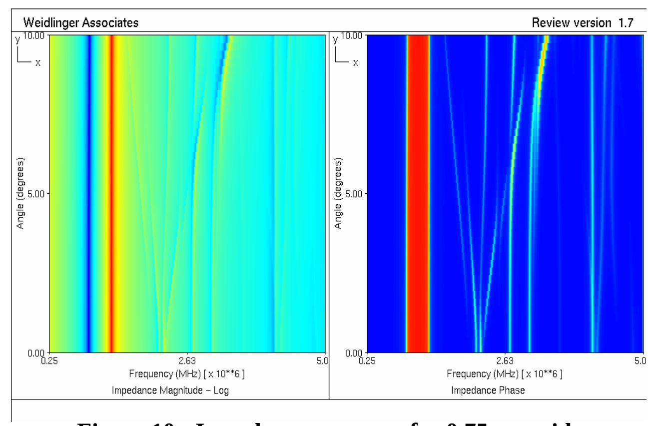

Figure 10: Impedance response for 0.75mm wide trapezoid (|Z| - left, ∠Z – right)

Height Skew – The Parallelepiped

In addition to the trapezoid described above, the parallelepiped structure shown in Figure 2 was also studied. The width of the samples was set to 0.75mm with a taper angle of 5°. Once again the height was 1.53mm and the length was 30.07mm. Figure 8 shows the correlation between experiment and simulation for the idealized, regular stair-step, fine stair-step and skewed model configurations. An initial look at these at these results would appear to suggest that even the idealized non-conforming grid demonstrates excellent correlation with experimental data. Indeed this is true, however on closer examination it becomes apparent that the small resonance at 1.6MHz has been missed entirely. On the other hand using stair-stepped or skewed meshes allows this mode to be predicted. The fine stair-stepped and skewed meshes provide the best correlation for all other resonances, including the mode at 4MHz, which is frequency shifted in the regular stair-stepped case. So once again, the skewed model provides the best correlation across the entire frequency band and is about 4 times faster than the fine stair-stepped calculation.

IV. Non-Idealized Structure and Spatially Variant Material Properties

In the previous section we demonstrated the improved level of correlation that was obtainable via the use of conforming skewed finite-element representations of a series of piezoelectric resonators. In this section we shall perform a more exhaustive analysis of how factors such as the degree of element taper and the spatial variability of material properties impact a transducer's resonant characteristics.

Structural Variation - Height taper (trapezoid)

The trapezoidal element shape shown in Figure 1 was used in a parameter sweep where the taper angle was swept from 0° to 10° taper in 0.25° steps. Element widths of both 0.5mm and 0.75mm were studied with the results being shown in Figures 9 & 10 respectively. Each figure shows the impedance magnitude (log scale) on the left and phase response on the right. The horizontal axis is frequency and vertical axis is angular

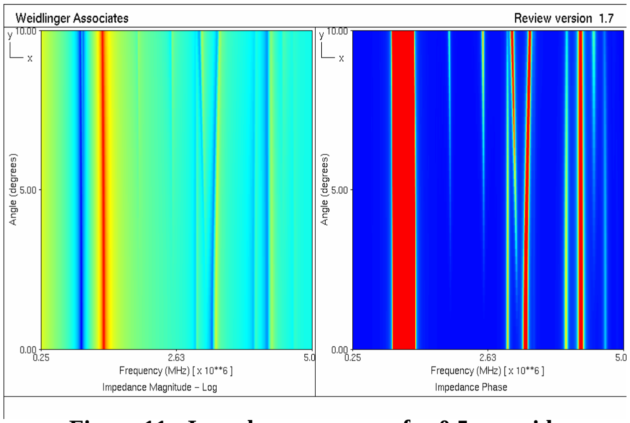

Figure 11: Impedance response for 0.5mm wide parallelepiped (|Z| - left, ∠Z – right)

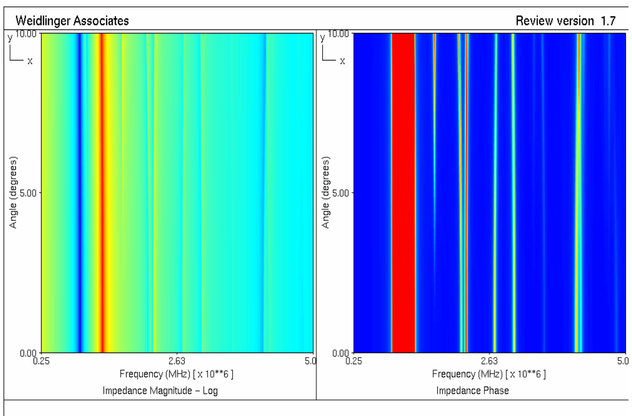

Figure 12: Impedance response for 0.75mm wide parallelepiped (|Z| - left, ∠Z – right)

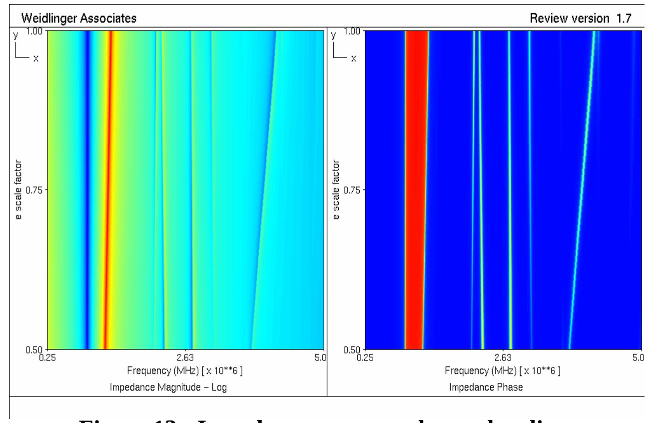

Figure 13: Impedance response due to depoling; 0.75mm wide (|Z| - left, ∠Z – right)

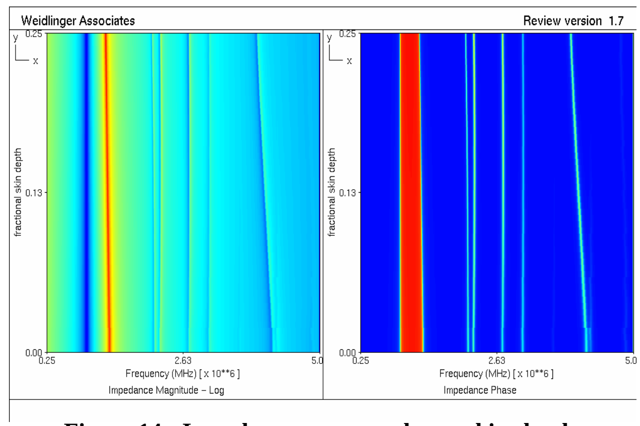

Figure 14: Impedance response due to skin-depth; 0.75mm wide (|Z| - left, ∠Z – right)

taper. It is obvious from Figures 9 & 10, that higher aspect-ratio (AR) devices are much more sensitive to increasing angular taper than is the case for elements with a lower AR. The lateral resonance that occurs at about 3MHz splits rapidly into two separate resonances as the taper angle increases and begins to interfere with the first harmonic of the thickness resonance (~4MHz) at a taper angle of about 5°. The 0.75mm device has a more strongly coupled response due to its reduced aspect ratio and hence the splitting and shifting of resonance modes it much more subtle for the same change of angular taper.

Structural Variation - Height skew (parallelepiped)

The parallelepiped element shown in Figure 2 was also used in a parameter study. The length and height were 30.07mm and 1.53mm respectively, the skew angle was varied from 0° to 10° in 0.25° steps. The impedance results for the 0.5mm and 0.75mm elements widths are shown in Figures 11 and 12 respectively. It may be clearly seen that the parallelepiped structure is much less sensitive to angular skew than the trapezoid is to the same amount of angular taper. At the higher skew angles, there some additional resonance activity does start to appear at about 1-1.5MHz, but it is only minimal.

From a designer's standpoint, the above results lead us to conclude that element taper must be more carefully controlled than element skew. Element skew could happen in practice if the piezoceramic substrate for an array was not mounted normal to the dicing blade. However, from the 0-10° range considered here, it can be seen that spurious resonant activity would be unlikely to arise in practical applications due to misalignment. What would be more likely is that an element's beam-pattern could be adversely impacted due to element skew. This issue however, will not be addressed within the scope of this paper.

Spatially Variant Material Properties

Not only does a resonator's physical geometry impact its resonance characteristics but also the level of homogeneity within the material will impact the sharpness of a device's resonances and will further cause a transducer's real-world performance to deviate from the ideal. With a view to further studying this issue, a series of simulations have been performed where the piezoelectric properties of the piezoceramic are changed through the resonator's width dimension. This is representative of depoling effects due to the dicing process.

Two parameter studies were carried out here. In both cases the resonator was 1.53mm and 0.75mm wide and had no taper or skew. In the first case, the skin-depth was kept constant at 15% of the element width and the piezoelectric coupling matrix **[e]** was scaled from 0.5 to 1.0 of its nominal values. Figure 13 shows the results of this variation with the vertical axis on both the magnitude response (left) and phase response (right) corresponding to the **[e]** matrix scale factor changing from 0.5 to 1.0. It may be seen from Figure 13 that the primary effect of reducing the piezoelectric coupling in the skin-depth is a reduction in the resonant frequency of the 1^{st} harmonic of the thickness mode. Figure 14 shows the effect of changing the skin-depth for the same structure. The **[e]** matrix scale factor was kept constant at 0.80 while the skin depth was varied from 0 to 25% of the element width. It may be clearly seen that the primary effect of changing the skin-depth is a small reduction in resonant frequency of the 1^{st} harmonic of the thickness mode.

The parameter variation used above is only a very small subset of the entire parameter space. It was not the intention of this work to infer that this particular type of variation in **[e]** is a accurate and complete quantification of what happens to a piezoceramic material due to work induced depoling. Most likely, there are also changes to the elastic and dielectric material properties. Furthermore, the simple multiplier scale-factor used here may be an over simplification. In reality, the various terms in the **[e]**, **[c]** and **[ε]** matrices may be all individually and uniquely scaled.

V. Conclusions

It has been shown that the level of correlation between experiment and theory can be improved by using finer stair-stepped or skewed finite element meshes. The improvement in correlation is dependant upon the physical structure being examined and the rate at which surface features change spatially. Furthermore, the use of a skewed finite element grid can offer improved computational efficiency and reduced run times for certain classes of structural geometry. This is not necessarily always the case and a transducer designer needs to aware of the pros and cons for each particular application.

It is apparent from the above results that changes to the lateral mode(s) are more prevalent with higher aspect ratio devices for a given angular taper. This is because the relative width change is greater for narrower elements. Consequently, from a manufacturing standpoint, if accurate control of lateral/higher-order modes is desired then greater attention to detail must be enforced during the fabrication process.

For the level of angular skew and taper seen in the experimental samples considered here, the differences in resonant characteristics are often subtle. So the question a designer has to pose is whether these subtle differences are relevant to the current design application. Before a final determination can be made, it would be necessary to establish whether these additional resonances and resonant shifts appreciably impact the device's pulse-echo response (spectral response & temporal ring-down) and beam-pattern characteristics. This is outside the scope of the current paper and will be looked at in greater depth at a later date.

VI. References

[1] N.N. Abboud et. al., "Finite Element Modeling for Ultrasonic Transducers" *Proc. SPIE International Symp. on Medical Imaging, Ultrasonic Transducer Engineering Conference*, K. Shung (ed), San Diego, Feb 21-27, (1998).