RESONANT CHARACTERISTICS OF PIEZOELECTRIC COMPOSITES: ANALYSIS OF SPURIOUS MODES IN SINGLE AND MULTI-ELEMENT ULTRASONIC TRANSDUCERS

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Abstract This paper examines in detail the generation of high frequency inter-pillar (lateral) resonances in 1-3 piezoelectric composites, through experiment and finite element analysis (FEA) with the PZFlex package, both in monolithically electroded composites and in multi-element arrays based on a 1-3 piezoelectric composite substrate. While thickness modes are accurately described by standing wave formation within the transducer, other inter-pillar modes will be clearly shown to be generated due to Lamb waves propagating through the substrate, between pillars and between elements in the single and multi-element cases respectively.

I. Introduction

Much work has been carried out in explaining the mechanisms of inter-pillar mode generation, and has concentrated on the behaviour of such modes in 1-3 composites. Auld [1] and Gururaja [2] compared the generation of inter-pillar resonances to Bragg scattering, usually found in crystal lattices. While still comparing these modes with Bragg scattering, Auld and Smith [3] stated that these resonances may be due to the formation of Lamb waves in the structure – however no significant work was carried out in relation to the type of Lamb wave generated, or the conditions under which it could be sustained. This paper represents an attempt to more completely describe the formation of these inter-pillar resonances by Lamb waves, with specific mention made of the type of Lamb wave generated and the Lamb wave velocity dependence on device microstructure. All FEA results presented in this paper are generated using the commercially available package PZFlex, version 1j7.

II. Single Element 1-3 Piezoelectric Composites

Using standard 'rules of thumb' a 1-3 piezoelectric composite was designed and manufactured with the specific aim of generating inter-pillar resonances. At 1.0 mm thick, a saw pitch of 0.73 mm, a kerf width of 0.30 mm, PZT5H piezoceramic and Ciba-Geigy CY1300/HY1301 polymer, this results in a 35% Volume Fraction (VF) transducer with a pillar Aspect Ratio (AR) of 0.43. PZFlex was used to simulate the electrical impedance of this device, and the comparison with experiment is shown in Figure 1, where all predicted frequencies are accurate to within 5%, less than the ceramic manufacturers specified tolerances.
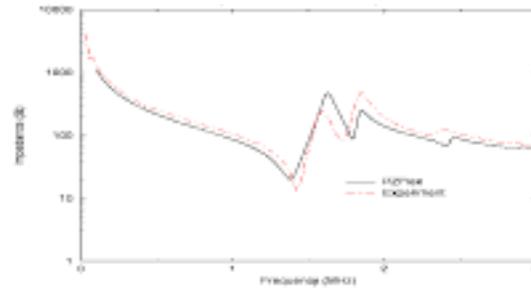

Figure 1: Electrical Impedance Magnitude of 1-3 Piezoelectric composite.

We can clearly see the thickness mode resonance at 1.42 MHz, with the first lateral resonance at 1.7 MHz and the second lateral resonance at 2.4 MHz closely impinging upon it.

In addition, the surface displacement at the inter-pillar resonances were predicted by PZFlex. These were then compared to those obtained experimentally via a scanning laser vibrometer, and an example result is shown in Figure 2a and 2b for the first inter-pillar resonance. An expanded 2 by 2 pillar section of the composite is shown, with polymer at the edges and very centre of the image. The PZFlex displacement plots also draw a border around the ceramic for simpler identification.

PZFlex is clearly predicting the surface displacements well. A distinct 'pattern' emerges, and differs for each of the lateral modes. By considering the nature of the piezoelectric composite we can surmise the mechanism by which these shapes are created. Imaginary 'lines of force' exist when the monolithically electroded 1-3 piezoelectric composite is activated and all the ceramic pillars compress and expand in phase. For the first inter-pillar resonance rows (or columns) of pillars act as such a
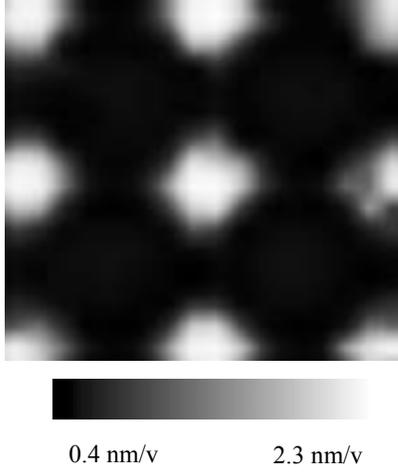
0.4 nm/v   2.3 nm/v

Figure 2a: First Lateral Resonance (1.7MHz) Experimental
             Displacement.
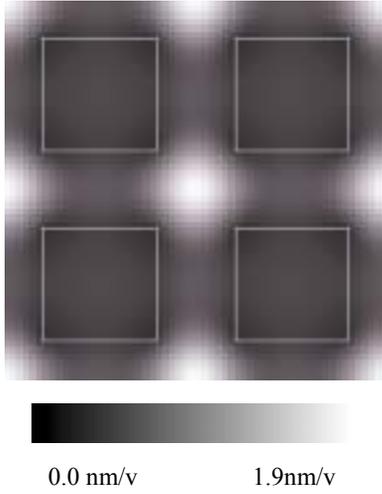

0.0 nm/v   1.9nm/v

Figure 2b: First Lateral Resonance (1.7MHz) PZFlex
             Displacement.

line, this requires a wavelength to equal d_0, the saw pitch. For the second inter-pillar resonance, although the distance between a ceramic pillar and its nearest neighbour on a diagonal is $\sqrt{2}d_0$, the distance between each 'finger' is $d_0/\sqrt{2}$ [4]. Thus the wavelength of the second inter-pillar resonance is $d_0/\sqrt{2}$. Obviously, additional 'fingers' would exist at right angles to those shown in the diagrams, that is, the structure is doubly periodic. It would be expected, therefore, that for the first inter-pillar resonance of a composite that waves would propagate normal to the transducer edges, while for the second resonance they would propagate in a 'diagonal' manner. As detailed by Viktorov[5], these lines of force will generate Lamb waves.

An additional complication of the Lamb waves is that their velocity (phase velocity v_{phase}), is dependent upon the type of Lamb mode and the frequency-thickness product (FTP)

of the plate. The FTP is defined as the product of the plate thickness with the operating frequency. Figure 3 shows the phase velocities for the two fundamental antisymmetric (a_0) and symmetric (s_0) modes for hardset epoxy – these are termed *dispersion curves*.
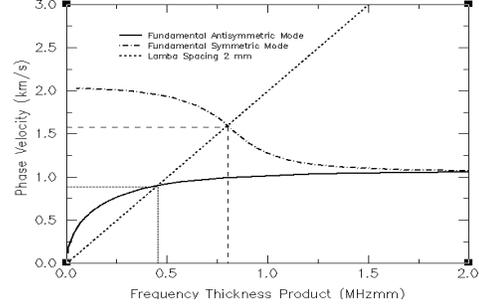

Figure 3: Dispersion Curve of Fundamental Modes for a
             Polymer.

The wavelength of a Lamb wave can therefore be expressed as Equation 1a

$$\lambda = \frac{v_{phase}}{f} \qquad\qquad v_{phase} = \lambda\, f \qquad\qquad \textbf{1a,b}$$

here λ is the Lamb wave wavelength, v_{phase} is the appropriate phase velocity, f is the operating frequency. The phase velocity of a material can be determined, for any particular wavelength, by drawing a line of the form Equation 1b.

The x-coordinate at which the line crosses the curve for each mode type (a_0 or s_0) is the FTP for that mode, while the y-coordinate is the phase velocity. Figure 3 demonstrates how this would be done for a 1mm thick Ciba-Geigy CY1301/HY1300 (hardset) polymer plate, with ID finger spacing of 2 mm. This arrangement would result in an a_0 mode of frequency 455 kHz and phase velocity 889 ms^{-1}, and an s_0 mode at 800 kHz with a phase velocity of 1580 ms^{-1}.

Since the wavelength is determined by the ID finger spacing (d_0 and $d_0/\sqrt{2}$), it is apparent that the first two mode frequencies generated by the Lamb modes will be

$$f_{L1} = \frac{v_{phase}}{d_0} \qquad\qquad f_{L2} = \frac{\sqrt{2}v_{phase}}{d_0} \qquad\qquad \textbf{2 a,b}$$

Rather than approaching the shear wave velocity of the medium, the phase velocity of the fundamental Lamb modes approaches the *Rayleigh* velocity at high values of FTP. An effective approximation for the Rayleigh wave velocity (v_R) can be found in Achenbach [6] and is typically 90-95% of the shear wave velocity. Rayleigh wave velocity is 1062 ms^{-1} , and shear wave velocity is 1150 ms^{-1} , for hardset epoxy.

An estimate of the phase velocity of a 1-3 piezoelectric composite can be made using the following equation

$$v_{phase} = v_{RPOLY} + (VF^2)(v_{RCER} - v_{RPOLY}) \qquad \textbf{3}$$

Where v_{RPOLY} is the Rayleigh wave velocity in the polymer, v_{RCER} is the Rayleigh wave velocity in the ceramic, and VF is the ceramic volume fraction, ranging from 0 to 1. This approximation will only hold true in cases where the Lamb wave propagates short distances. Note that the phase velocity calculated is always greater than the lowest velocity in the constituent materials. This 'effective v_{phase}' is only valid for inter-pillar modes in piezoelectric composites, where each Lamb wave propagates a single wavelength before being 'reinforced' by the action of an adjacent ceramic pillar. In cases where the Lamb wave travels multiple wavelengths the effective phase velocity is likely to be reduced by the mass loading of the ceramic on the polymer – a condition not present in uniformly electroded 1-3 piezoelectric composites. Given the entirely symmetrical structure and drive conditions of the piezoelectric composite, the Lamb wave can only be the fundamental symmetrical wave (s_0).

It is important to note that the 'driving force' behind the inter-pillar resonances is the thickness mode displacement of the piezoelectric composite. The inter-pillar resonances require this displacement to be generated, and so while these resonances will always exist, they will only become prominent when their frequencies approach those of the fundamental thickness mode and its harmonics.

III. Multi-Element 1-3 Piezoelectric Composites

While single element piezoelectric composite devices have been made for many years, the last decade has seen a number of array devices manufactured from 1-3 piezoelectric composites by using the electrode pattern to define the array elements. This offers advantages in simplicity of design, but often higher frequency resonances occur which disrupt beam patterns, and consequently require structural adjustments such as sub-dicing to retain effectiveness.

The investigation into the cause of resonant modes within piezoelectric composites was therefore extended to this class of devices. An example of a piezoelectric composite and array electroding can be seen in Figure 4. The active array element is thus determined by application of voltage to a specific electrode, and leaving all other electrodes open. Figure 5 details the impedance magnitude plot of a 35% PZT5H/Hardset epoxy piezoelectric composite substrate (monolithically electroded), both experimentally and as modelled by PZFlex.

It can be clearly seen that there is excellent correlation between experiment and PZFlex, and that the base substrate is unimodal. To determine the inter-element

coupling between array elements, a scanning laser vibrometer was used to obtain the displacement across all elements when the centre element #4 was activated. Results can be seen in Figure 6.
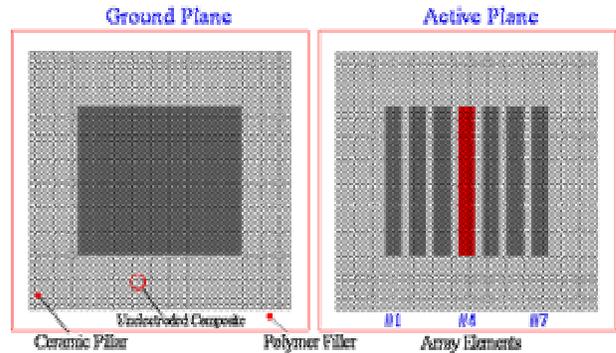

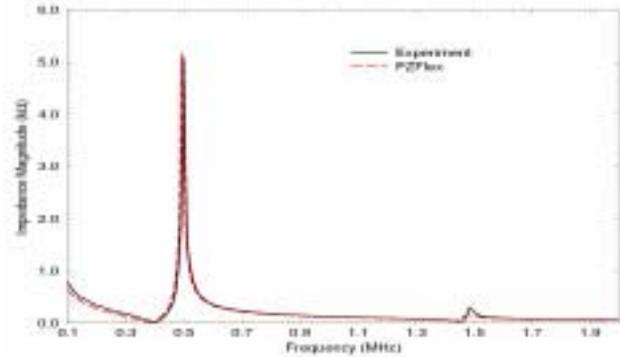Figure 4: Array Defination by Electrode on 1-3 Piezoelectric composite.

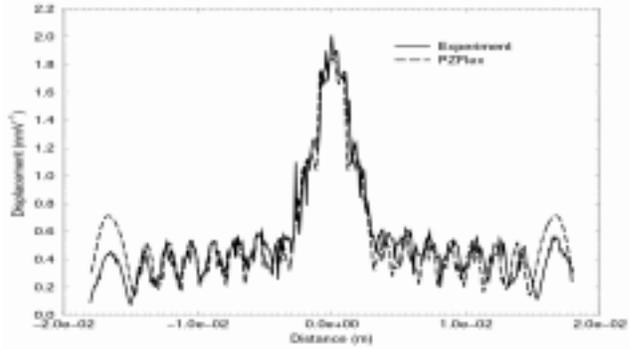Figure5: Electrical Impedance Magnitude for PZT5H/Hardset 1-3Piezoelectric composite.

Figure 6: Surface Displacement Across Array Elements on Device with Hardset Epoxy.

Once again we can clearly see the good correlation between experiment and PZFlex, with strong inter-element coupling evident in the hardest epoxy device. This is due to the low damping in the epoxy failing to attenuate the laterally propagating waves that induce the unwanted activity.

These additional modes were suspected to be due to Lamb waves induced by the element definitions themselves. To investigate, the experimental apparatus allowing experimental generating and detection of Lamb waves in a piezoelectric composite structure by a combination of laser excitation of waves, and a laser vibrometer to capture vibration data, was utilised. By application of a broadband pulse, all modes should be generated within the structure, and by sampling the displacement response across the surface, a 2D FFT can be applied to the results to determine the dispersion curves for the sample.

This can also be simulated by the PZFlex finite element code. A comparison of 3 dispersion curves, experimental, PZFlex, and analytically calculated are presented in Figure 7 for an aluminium plate. As can be seen, excellent correlation exists between all methods. The analytical method is only truly applicable in the case of a homogenous material.

This method was then expanded to calculate the curves for a 1-3 piezoelectric composite substructure. Figure 8 shows the results when this was applied to a 35% VF PZT5H piezoelectric composite. Both experiment and PZFlex predict the existence of Lamb waves within the structure which will contribute to poor device behaviour. Interestingly, the velocity of these Lamb waves is lower than that of the slowest wave in the constituent materials. It appears that unlike in the monolithically electroded case, the waves are propagating large distances and so the ceramic pillars effectively 'mass load' the polymer and reduce wave velocities. Thus while the mechanism of higher order resonance generation is identical in single and multi-element 1-3 piezoelectric composites, the phase velocity that determines frequency is dependent upon both constitutive materials and the volume fraction, and the array definition.

IV. Conclusions

1-3 connectivity piezoelectric transducer structure support Lamb wave mode generation, in particular the s_0. These modes may be identified from the element impedance response, measured experimentally or predicted using FE methods. In multi-element piezoelectric composite structures, the propagating mode travels at a velocity below that of the constituent material with the lowest velocity, while in monolithic transducer array structures, Lamb wave propagation at a velocity between that of the two component materials. The lateral modes thus induced only couple strongly when their frequency is approximate to that of a strong thickness mode or its harmonics. Both element spacing and pillar spacing, along with material properties, are critical to the frequencies at strengths at which these Lamb wave induced modes occur.

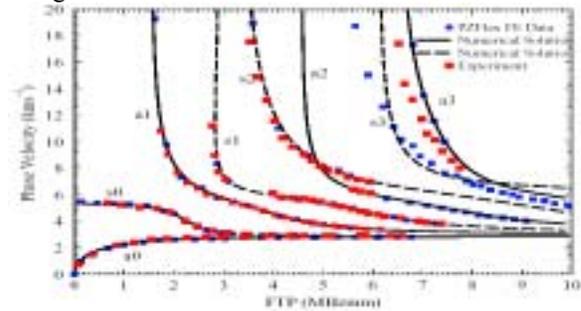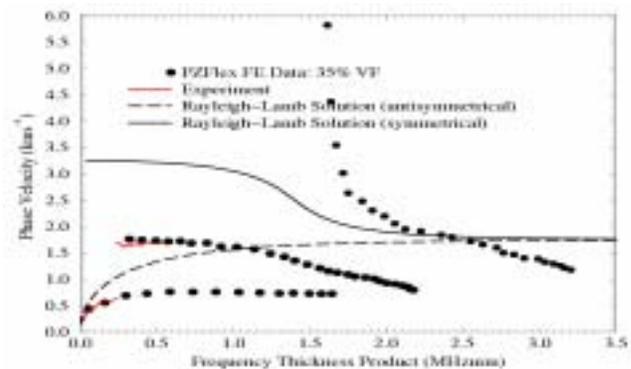Figure 7: Dispersion Curves for Aluminium.

Figure 8: Dispersion Curve for a 35% VF Piezoelectric composite.

VI. References

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