Finite-Element Determination of Electromechanical Coupling Coefficient with Applications to Piezoelectric and Electrostatic Transducers

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Purpose

- Aid transducer designer
- Medical ultrasonic imaging requirements:
  - high bandwidth and sensitivity
  - small size
  - high uniformity
  - cost constraints
- Must evaluate new technologies against current methods

Example Application

- Low-MHz frequency, 64-256 element phased, linear, and curved linear arrays
- Currently stacked PZT ceramic with two matching layers and lens
- Fine dimensions, expensive and somewhat fragile

Example Application

- Design sophistication has progressed to FEA
  - Must include 3D effects
  - FDA limits on power affect performance
- Bandwidth strongly affects performance as well
  - Related to coupling coefficient

Example Biomedical Image

Coupling Coefficient - k

- Efficiency of input electrical energy to output mechanical energy (& vice-versa)
- Valid for any 2 port transducer
  - k^2 is the fraction of the energy stored when input from one port is available at the other port.
  - Tuned bandwidth is proportional to k (Fano's theorem)

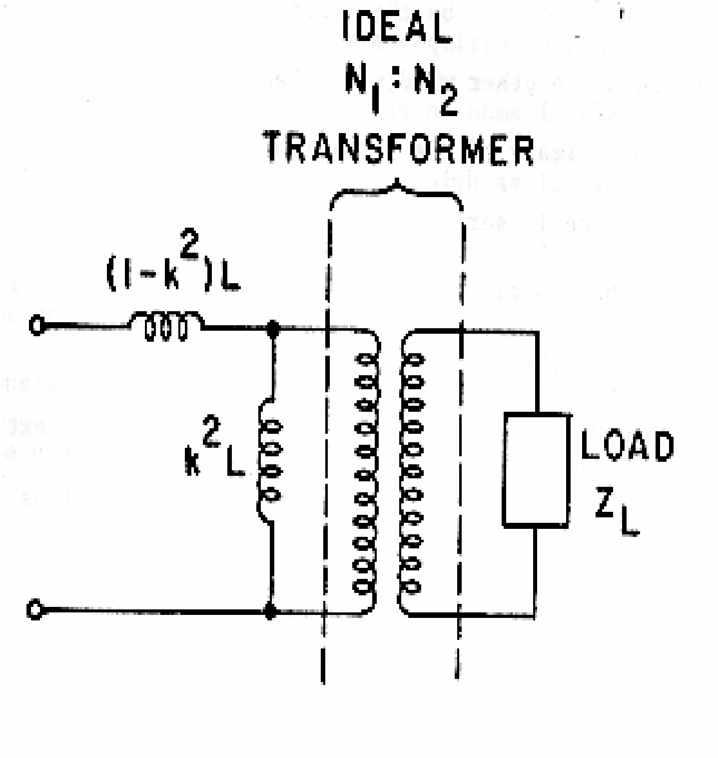
Example of k

Non-ideal electrical transformer.

Energy is stored in magnetic flux due to current in one winding.

A fraction k of that flux passes through the other winding.

The fraction of the total energy stored which is mutual is k^2.

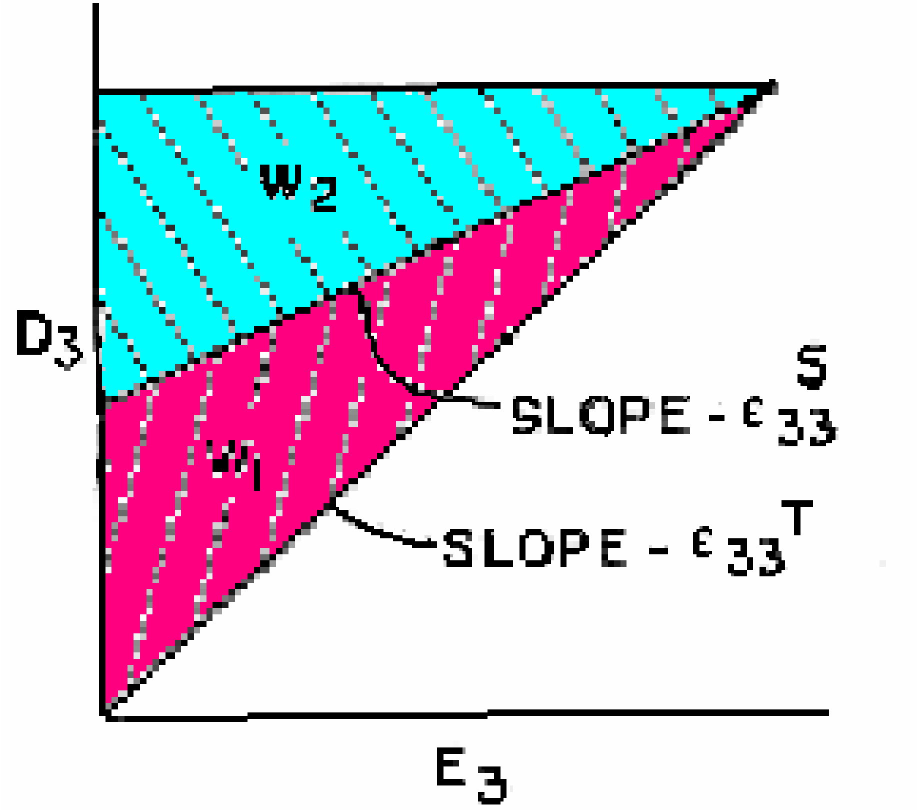


Calculating k for Piezoelectrics

- k_T is the thickness mode coupling
- Approximation available using resonant frequencies
  - Valid for unimodal, loss-free device, operating in air
  - Measurement of dielectric, electromechanical and elastic energies
  - Impractical

Calculating k for Piezoelectrics

- Drive a piezoelectric material to a specific electric field with the surfaces free
- Fix the surfaces and drive the field back to its original value
- Discussed by Berlincourt in Mattiat



Calculating k for Piezoelectrics

- ‘Blue’ energy is mechanical (mutual)
  - not recovered
  - ‘Red’ + ‘Blue’ energy is total energy stored
  - total input energy
  - k^2 = ‘Red’ / (‘Red’ + ‘Blue’)
- $k^2 = (\varepsilon^T - \varepsilon^S) / \varepsilon^T$

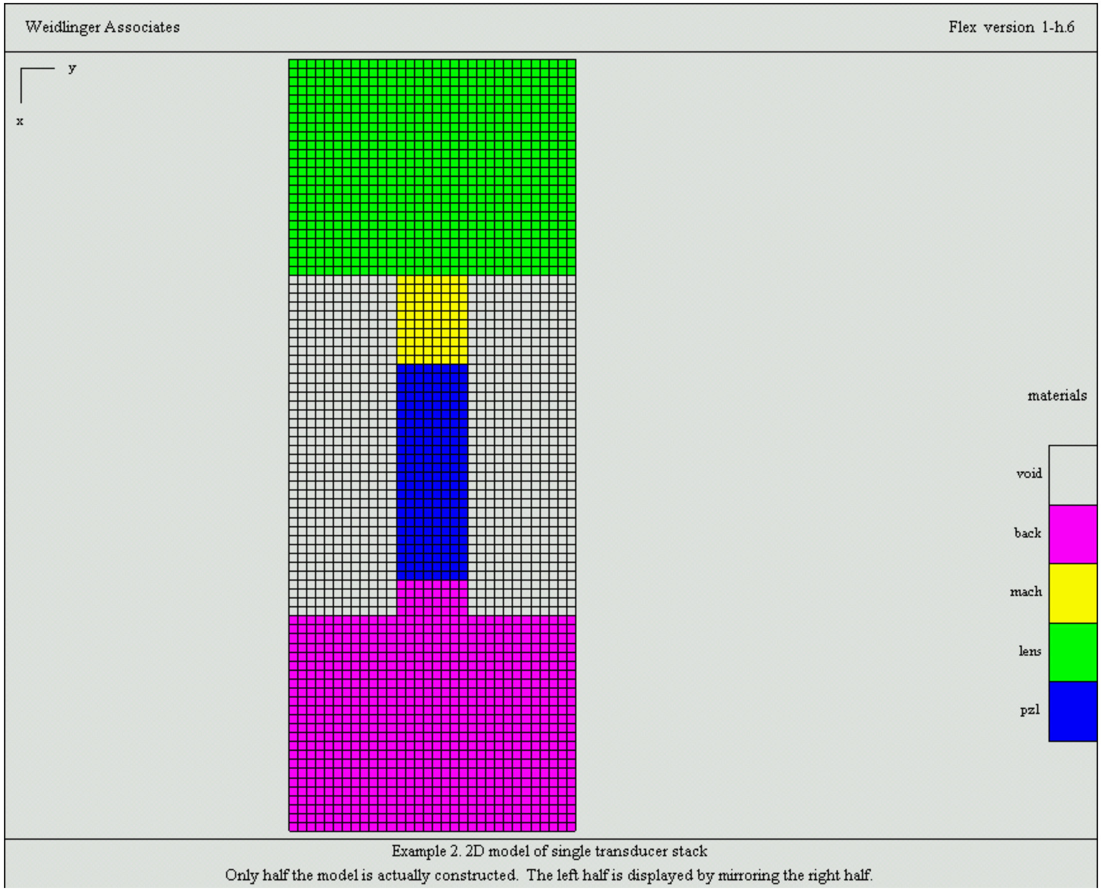
Performing Experiment

- Difficult experiment to perform
  - Requires almost perfect control of conditions
- Finite Element Analysis
  - Allows perfect control of conditions
  - Perfect reproducibility
  - Exact measurements
  - ‘Virtual’ experiment

What is FEA?

- FEA stands for Finite Element Analysis
- A system is broken into discrete parts called *elements*
- Each element has its own governing equations
- Apply boundary conditions and solve – ideal for computers
- Some approximation always required

Example FEA Mesh



Example 2. 2D model of single transducer stack

Only half the model is actually constructed. The left half is displayed by mirroring the right half.

What is FEA?

- Predicts system behaviour accurately if done properly
- Affordable software and hardware now powerful enough to model complete systems
- Fewer approximations needed
- *Virtual Prototyping* now practical

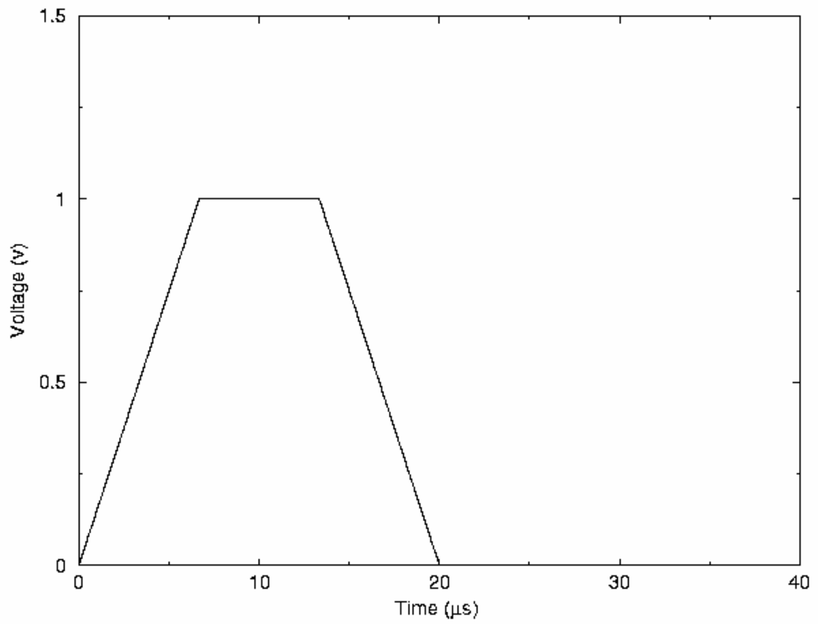
Sample PZFlex Results

- Comparison of PZFlex Results to values in 'Kino', 'Acoustic Waves - Devices, Imaging and Analog signal Processing'

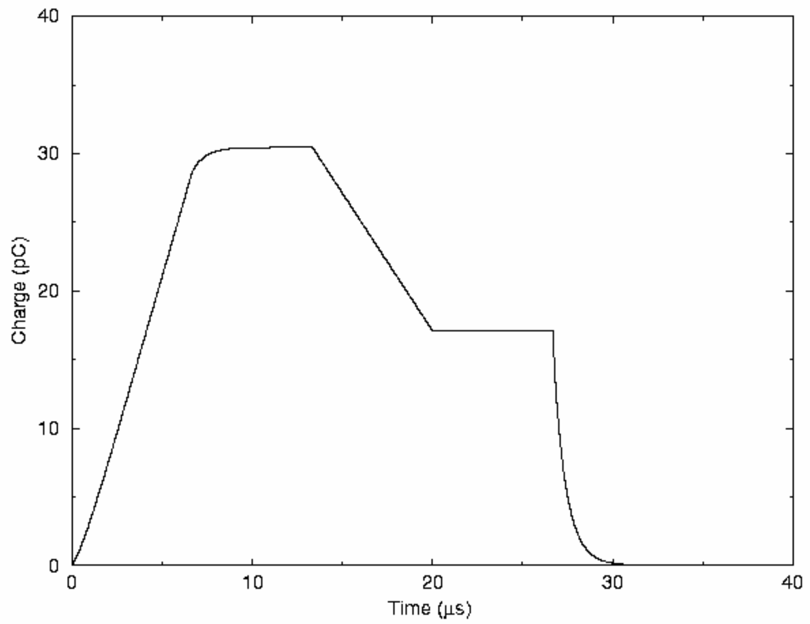
	PZFlex	Kino	PZFlex	Kino
Ceramic	k_T	k_T	k_{33}	k_{33}
PZT4	0.51	0.51	0.724	0.70
PZT5A	0.485	0.49	0.701	0.705
PZT5H	0.51	0.505	0.747	0.75
PZT8	0.47	0.48	0.634	0.64

Piezolectric Material Charge Response

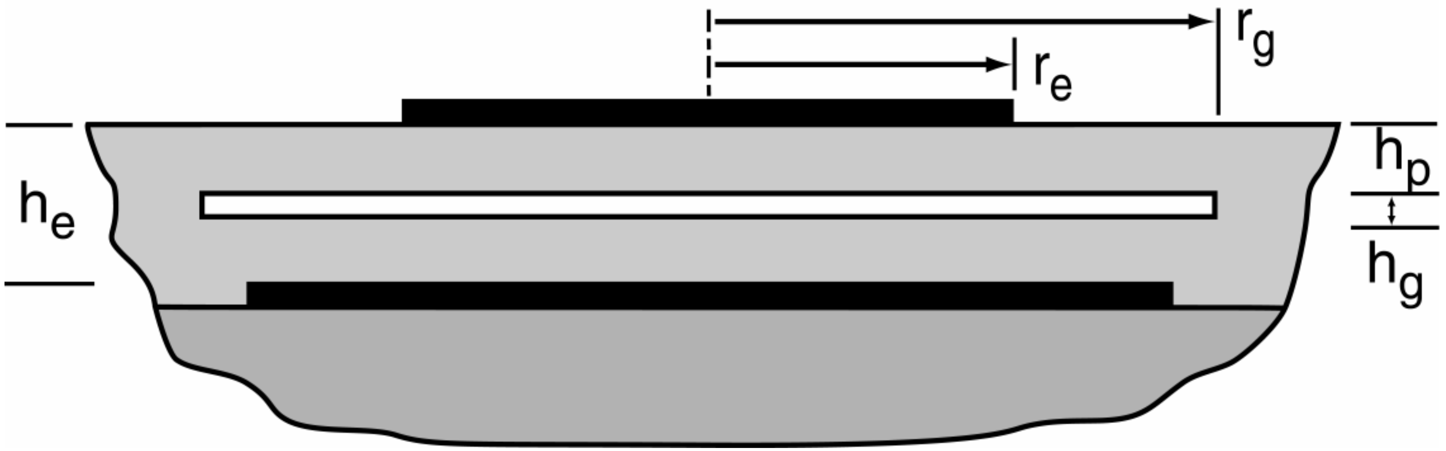
Voltage Applied to Piezoelectric Material



Charge Developed on Piezoelectric Material



MUT Schematic



r_g: gap radius
r_e: electrode radius

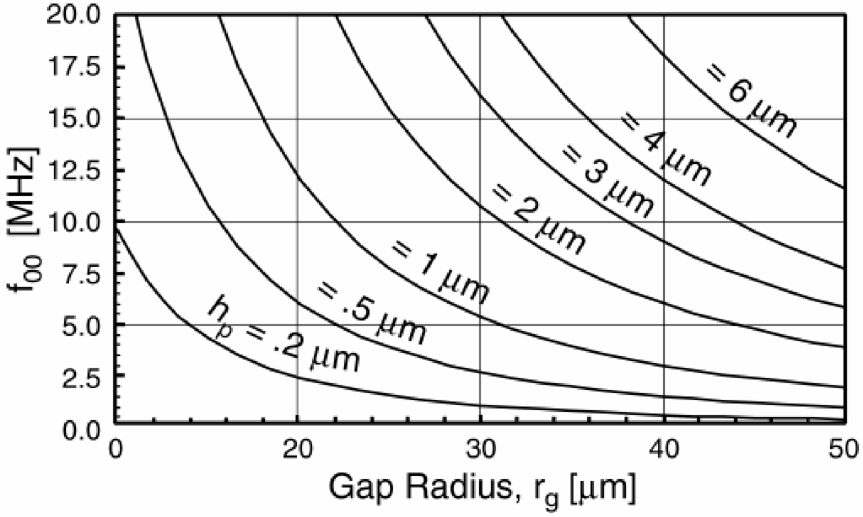
h_e: electrode spacing
h_g: gap thickness
h_p: plate thickness

MUT Construction

- Micromachined Ultrasonic Transducer
- Pioneered at Stanford University in this form
- Manufactured using semiconductor processes
  - Large scale for semiconductors, so tolerances very fine, well tested technology
  - Device integration, mass production
  - 2D array construction simplified
- Use electrostatic forces between two plates to transmit and receive ultrasound

MUT Operation

- Require biasing to operate
  - Nonlinear response
- Basic operation well understood



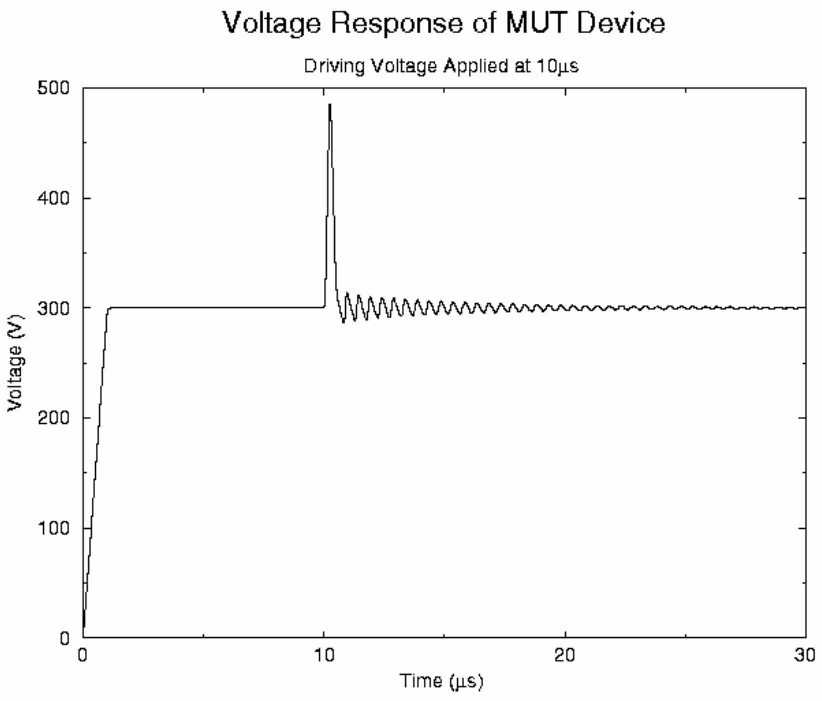
$$f \approx \frac{t}{r^2}$$

f - fundamental frequency

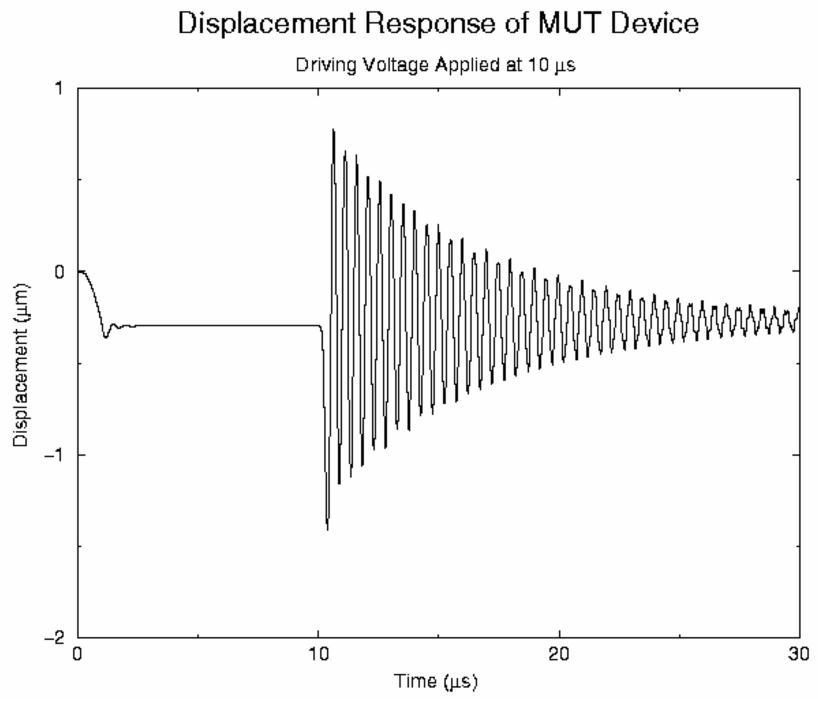
t - membrane thickness

r- membrane radius

Voltage and Displacement Response for MUT


Voltage Response of MUT Device

Driving Voltage Applied at 10µs


Displacement Response of MUT Device

Driving Voltage Applied at 10 µs

Comparing Electrostatics to Piezoelectrics

- Consider charge rather than displacement, voltage rather than electric field, and total energy rather than energy density.
- Capacitance replaces dielectric constant. If identical voltage conditions are used for fixed and free cases, charge may be used.

Virtual Prototyping

- New considerations for this technology
    - Nonlinear response complicates analytical models
    - Harmonic distortion possible
    - Rayleigh wave generation in silicon backing
    - Residual stress in membrane from manufacturing
    - Not as ‘mature’ as piezoelectric devices
    - Need to relate to existing devices
- Finite Element Modelling used for analysis
    - PZFlex adapted for electrostatic simulations

Advantages of PZFlex

- Extensive validation exercises performed
  - Utilised in many biomedical companies and naval facilities for ultrasonic device modelling
- Explicit modelling approach
  - Allows larger models than traditional approach for given memory
  - Reduces computation time for typical problems
- Transient analysis
  - Time domain code allows for broadband analysis
  - Also allows for nonlinearities to be considered

Electrostatic Modelling Considerations

- Device scale disparity
  - Device thickness: ~2μm, width > 50 μm, λ in water at 1MHz: 1.5mm
  - Shell elements utilised
- Device capacitance and membrane stiffness alters with displacement
  - Time domain PZFlex determines new capacitance and stiffness each time step - no approximations needed
- ‘Non-zero’ initial conditions
  - Code allows for moving to biased and pre-stressed position before excitation function applied

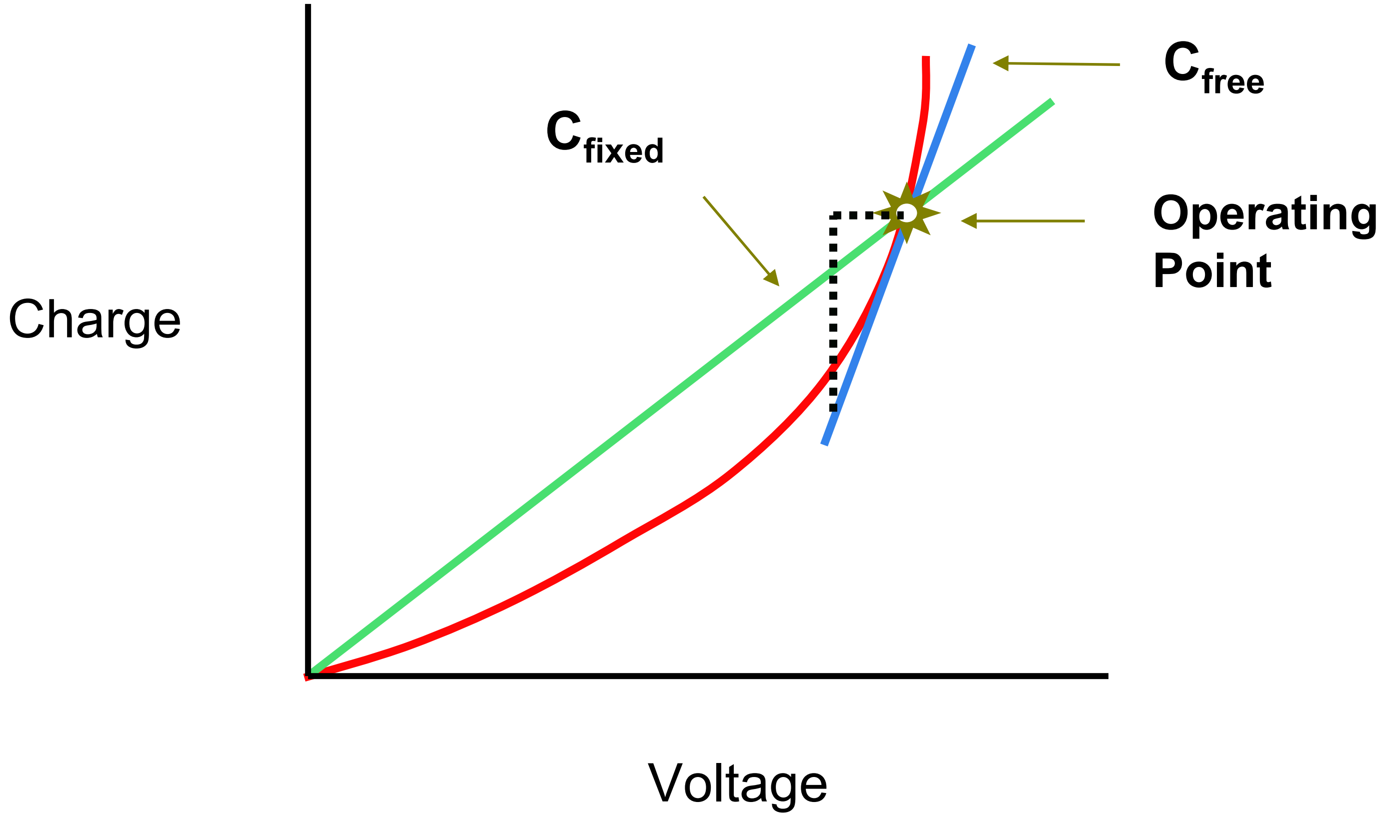
Generalized Berlincourt Coupling Definition for Electrostatic Devices

- With the device free to move, measure the static charge or capacitance, C^T
- Repeat with the device fixed to obtain C^S
- $k^2 = (C^T - C^S)/ C^T = 1 - C^S/C^T$
- Applicable to cMUTs
  - Care must be taken in appropriate boundary conditions

cMUT FEA Coupling Determination

- Coupling is function of bias voltage
  - Multiple readings taken at various bias levels
  - PZFlex requires only one simulation for all
- F proportional to Q^2
  - Eventually overcomes strength of material
  - Collapse
- Charge voltage curve starts as linear
  - then rapidly increases rate of change

Capacitance of cMUT under Various Conditions



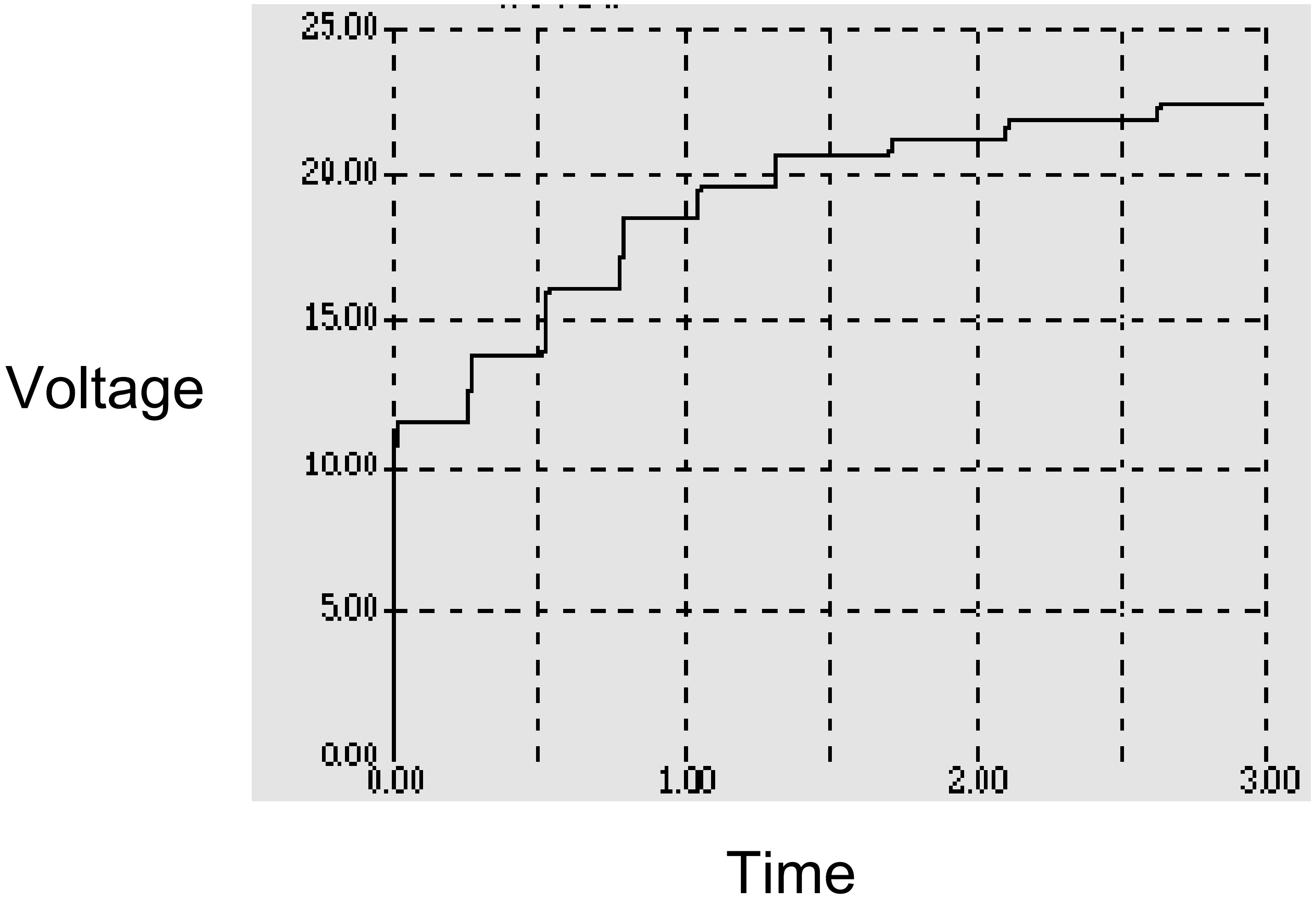
Capacitance of cMUT

- Slope of the Q-V curve is the free capacitance
  - Slope from origin to operating point is the the fixed capacitance
  - Coupling can approach 1 as collapse is approached
- Parasitic series capacitance undesirable
  - lowers maximum achievable coupling coefficient by making the free capacitance at collapse finite

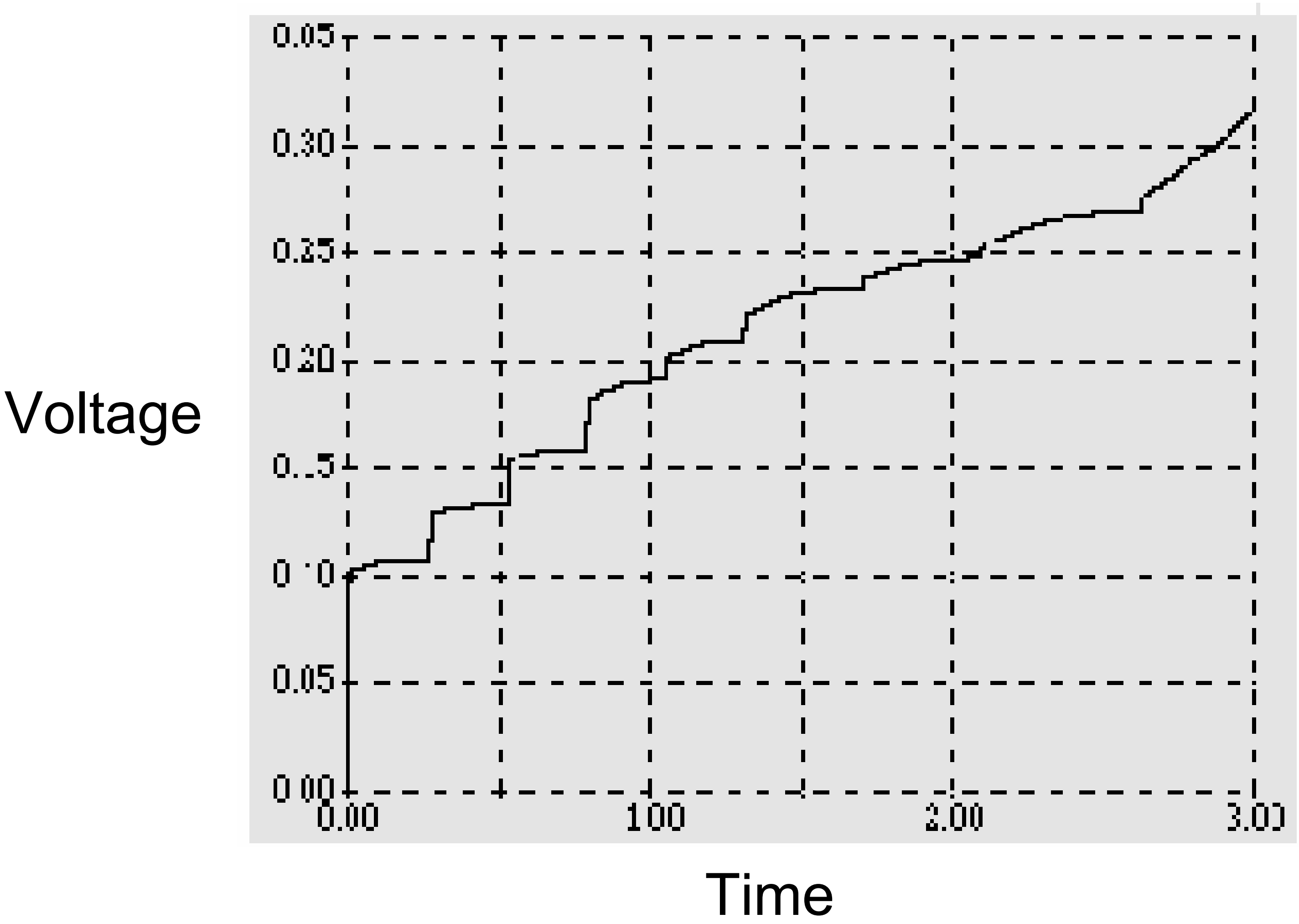
PZFlex Results

- Voltage was stepped faster than the membrane could respond
  - allows determination of the fixed capacitance
- Membrane allowed to move to equilibrium
  - free capacitance determined
  - $k^2 = 1 - Q_{fixed}/Q_{free}$.

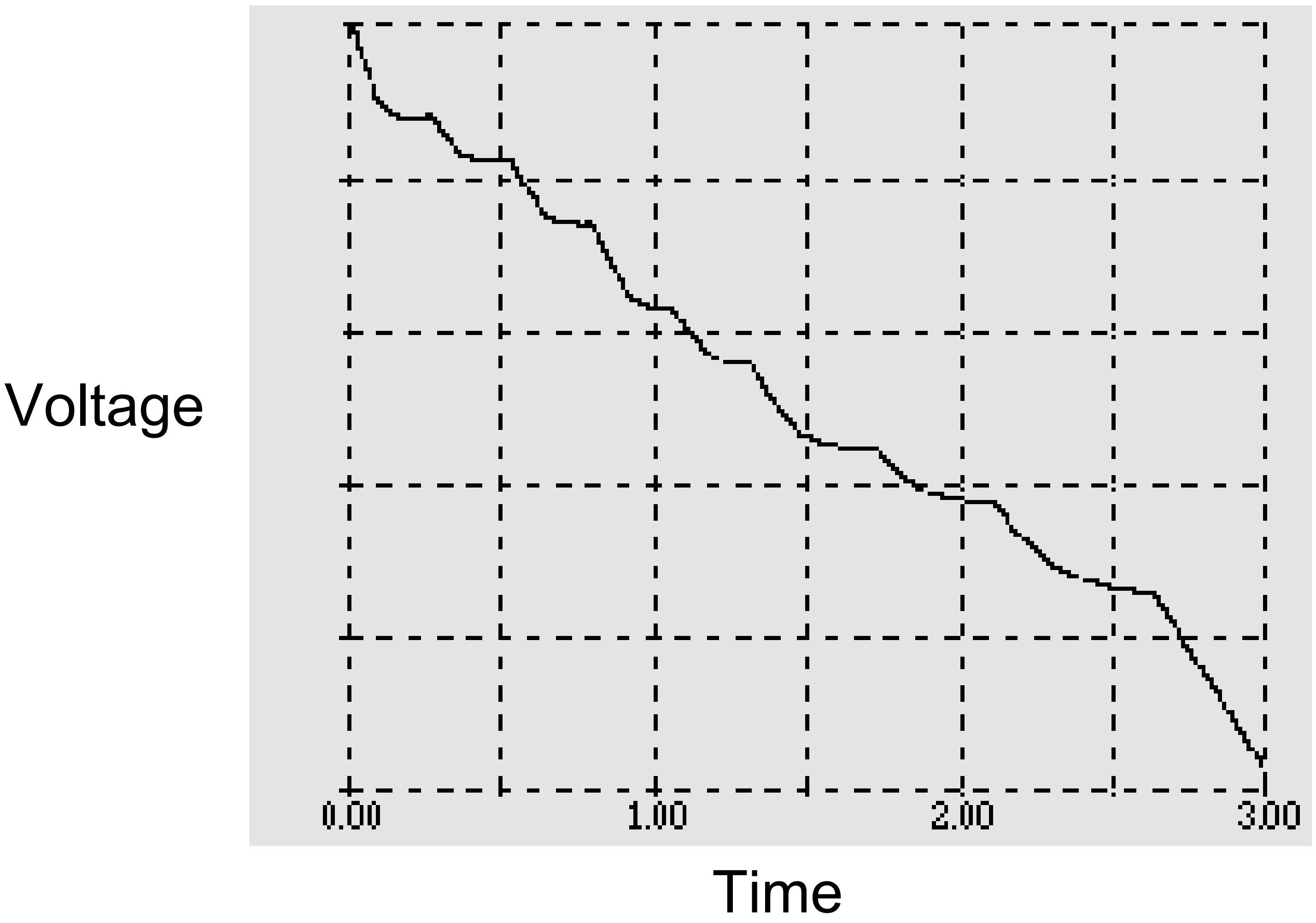
Voltage -Time Response



Charge-Time Response



Displacement-Time Response



PZFlex Results

- More time is necessary to reach equilibrium as the collapse point is approached.
  - Effective stiffness of the membrane is determined more and more by the electrical load.
  - Load is short circuit, equivalent to a soft mechanical surface.
- For a cMUT at high coupling, the acoustic input impedance will be determined by the electrical load, which must therefore be controlled.

PZFlex Results

- Bias voltage is increased
  - Coupling increases from .05 to over .65 before the collapse is reached.
- The difference in voltage between k^2 of .65 and collapse is less than 5%.
  - Due to the parasitic series capacitance incorporated into this model

Further Work and Considerations

- PZFlex demonstrates further useful design tool
  - Rapid determination of k under various conditions
  - Collapse voltage, stresses, parasitic modes etc can be fully considered
- Determination of ideal operating point
  - Tradeoff drive against bias
- Reduce parasitic capacitances
  - Good design
  - New materials

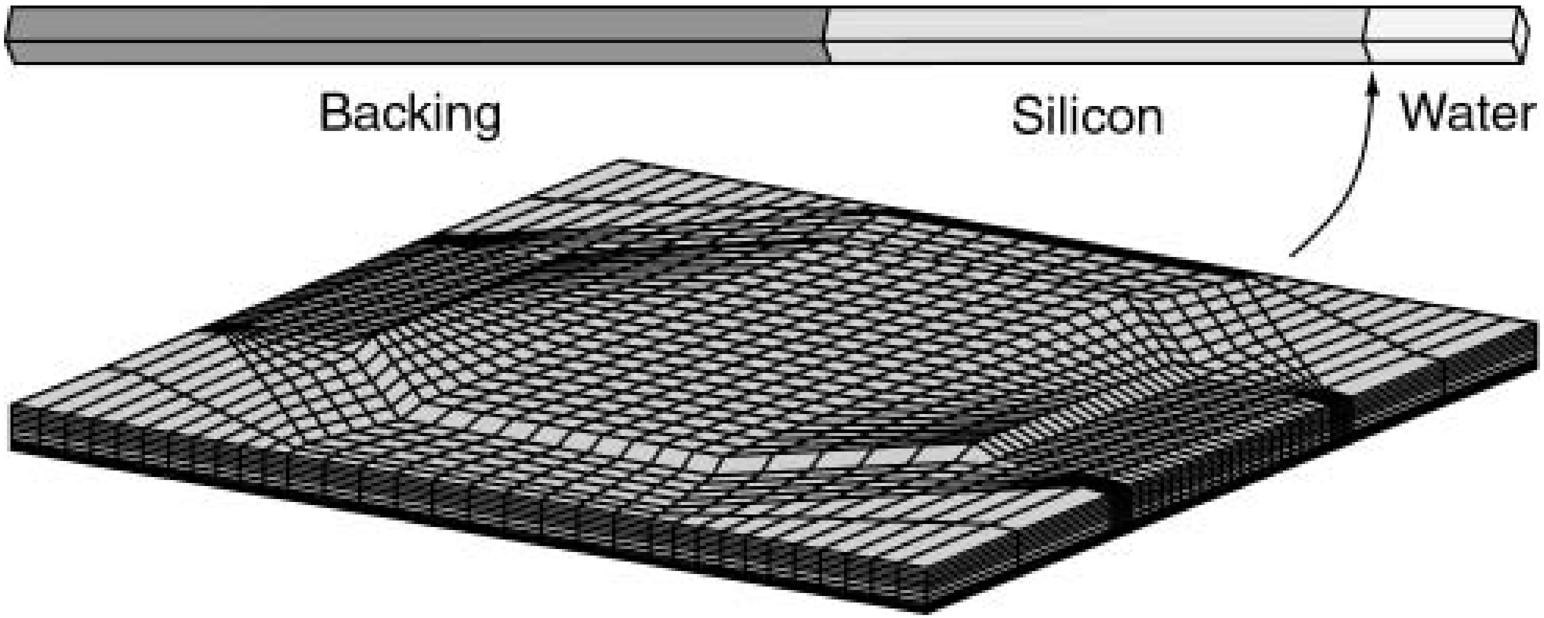
Further Work and Considerations

- Material tolerances to be investigated
  - Manufacturing techniques adequate for biomedical applications
- cMUTs still improving
  - Potentially cheaper, more robust and versatile than current medical arrays

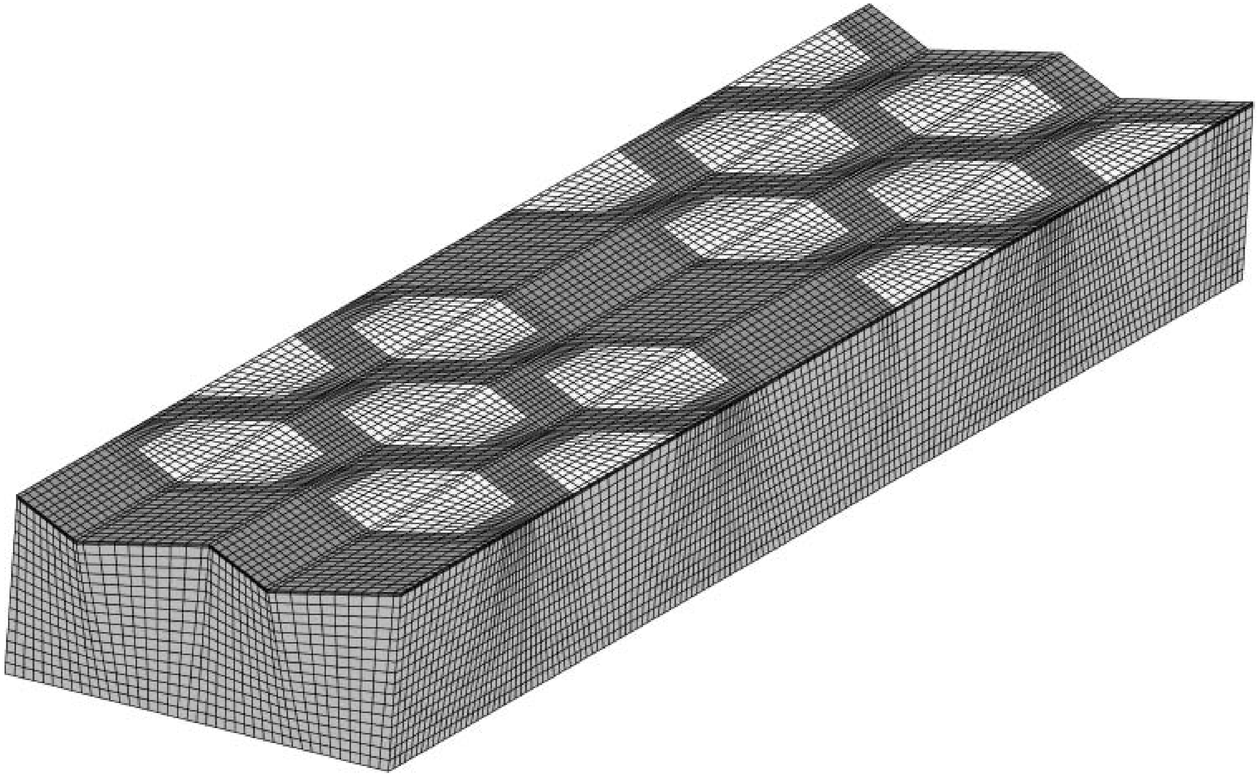
Validation Exercise

- Experimental results provided by Sensant
  - 7.5 MHz device
  - Early results show extensive ringing
- Device modelling with PZFlex show Rayleigh wave in silicon backing
  - Addition of matched, lossy, backing removes wave
- Resonant waves in silicon unlikely to be significant in lower frequency naval devices

“Unit Cell” Model of Sensant Array



Section of Large-Scale MUT Model



Insertion Loss from PZFlex Versus Sensant Data